MTL106 Probability and Stochastic Processes

Sem-II 2016-17

TUTORIAL-4 Moment Generating Function

Solutions

1. Var(X) = E[ (X – E[X])2 ]

Since (X – E[X])2 ≥ 0, it implies that E[ (X – E[X])2 ] ≥ 0

Hence Var(X) ≥ 0

Var(X) = E[ (X – E[X])2 ] = E[ X2 + E2[X] – 2\*X\*E[X] ]

= E[X2] + E[ E2[X] ] – E[ 2\*X\*E[X] ]

= E[X2] + E2[X] – 2\*E[X]\*E[X]

= E[X2] + E2[X] - 2E2[X]

= E[X2] - E2[X] ≥ 0

Hence E[X2] ≥ E2[X]

1. MY(t) =

MY(t) = E()

P(Y = -3) =

P(Y = -1) =

P(Y = 0) =

P(Y = 2) =

Also so it is a valid pmf

1. a. MW(t) =

MW(t) = E[etW]

PW(W = 4) = 1

So P(1 < W ≤ 5) =1

Hence, False

b. Var[X] = E[X2] – E2[X]

E[X2] =

=

=

This does not converge. Since E[X2] does not exist, variance does not exist. Hence False

1. Z = X1+X2+…+X100/100

MZ(t) = E[] = E[] = E[] ------ (i)

Since Xi’s are independent, so are

We will first prove that if A and B are two independent random variables then E[A.B] = E[A].E[B]

E[A.B] = =

(Since A and B are independent fA,B(a,b) = fA(a).fB(b))

= = E[A].E[B]

Using this in (i), we get MZ(t) = E[]… ]

= MX(t/100)…MX(t/100) [Since Xi are iid their MGF are all same]

=

1. E[X] = 1 and E[X2] = 1

Var(X) = E[X2] – E2[X] = 1 – 1 = 0

This means that X is a constant random variable with P(X = 1) = 1

1. E[ (X – E[X])4 ] = 1\*( (1-1)4 ) = 0
2. P(< X ≤ 3) = 1 since 1 lies in this interval and P(X = 0) = 0
3. MX(t) = exp(µ())

Put t = 0, we get E[X] = µ

Put t = 0, we get E[X2] = µ2 + µ

Var[X] = µ2 + µ - µ2 = µ

As mean and variance are same, we get a hint that it may be poisson distribution

Let us find MGF of poisson distribution with parameter µ

MPoi(t) =

=

=

=

=

So X is poisson distributed with parameter µ

P( 4 – 2\*2 < X < 4 + 2\*2) = P( 0 < X < 8 )

= P( 1 ≤ X ≤ 7 )

=

1. fX(x) =

MX(t) =

=

=

=

=

(The integral evaluates to 1 as it is integrating pdf a normally distributed random variable with mean σ2t and variance σ2 over entire real line)

MX(t) = = +

nth order moment about zero is

When n is odd, after taking derivative n times, sum comprises of only odd powers of t with no constant term so put t = 0 makes the sum 0.

When n is even, after taking derivative n times, we get a sum having following form …

When t=0 is substituted only a1 remains so nth order moment when n is even is a1

1. MX(t) =

From observation,

P(X = 0) = , P(X = -1) = , P(X = 1) =

µ = 0\*(1/6) + (-1)\*(1/2) + 1\*(1/3) = -(1/6)

σ2 = E[X2] - µ = 02\*(1/6) + (-1)2\*(1/2) + 12\*(1/3) – (-1/6)2= 1/2 + 1/3 =5/6 – 1/36 = 29/36

P(-1/6 – /6 < X < -1/6 + ) = P(-6.3/6 < X < 4.3/6) = P(-1.05 < X < 0.71)

= P(X=-1) + P(X = 0) = 1/2 + 1/6 = 2/3

1. MX(t) = E[] = =

=

=

=

=

[Converting into = form with n=2 and x = -2t]

=

=

1. E[Xn] = n!

MX(t) = E[] = =

=

=

= , |t| < 1

1. Let X and Y be two independent poisson distributed random variables with parameter λ=1

MX(t) = MY(t) =

Z =X+Y and we need to find mgf for Z

MZ(t) = E[] = E[] = E[]

As X and Y are independent, so are expectation of their product is same as product of their expectation

MZ(t) = =

Hence Z is a poisson distributed random variable with parameter λ=2

MX(t) = E[]

=

=

= 2